CHAPTER ONE

Indices Exponential Equations And Application Of Algebra

Indices:

1. a^n x a^m = a^{n+m}
   This is the first law of indices.
2. $a^n \div a^m = a^{n-m}$
   This is the second law of indices.

   Examples:

1. 2^3 x 2^4 = 2^{3+4} = 2^7
2. 2^6 x 2^4 = 2^{6+4} = 2^{10}
3. 3^4 x 3 = 3^4 x 3^1 = $3^{4+1} = 3^5$
4. 3^6 x 3^2 = 3^{6+2} = 3^8
5. 2^3 x 2^4 x 2^2 = $2^{3+4+2} = 2^9$
6. 4^3 x 4 x 4^2 = 4^3 x 4^1 x 4^2 = 4^{3+1+2} = 4^6
7. $2^6 \div 2^4 = 2^{6-4} = 2^2$
8. $3^8 \div 3^2 = 3^{8-2} = 3^6$

Simplify the following:

1. $\frac{2^3 \times 2^6}{2^4}$

Soln.

$\frac{2^3 \times 2^6}{2^4} = \frac{2^{3+6}}{2^4} = \frac{2^9}{2^4} = 2^9 \div 2^4 = 2^{9-4} = 2^5$.

2. $\frac{3^4 \times 3^6}{3} = \frac{3^4 \times 3^6}{3^1} = \frac{3^{4+6}}{3^1} = \frac{3^{10}}{3^1} = 3^{10-1} = 3^9$.
3. $\frac{2^4 \times 2 \times 2^3}{2^6 \times 2^2} = \frac{2^{4+1+3}}{2^{6+2}} = \frac{2^8}{2^8} = 2^{8-8} = 2^0 = 1$

N/B: Any number raised to the power zero = 1.

4. 9^{a^2} x $3^{a^{-6}}$ = 9 x 3 x a^2 x a^{a-6} = 27 x a^{2+-6} = 27x a^{2-6}
   = $27a^{-4}$
5. a^{-4} x a^{-2} = a^{-4+-2} = a^{-4-2} = a^{-6}

6. $3a^{-4}$ x $2a^{-3} = 3 \times 2\ a^{-4} \times a^{-3} = 6 \times a^{-4+-3}$
$= 6a^{-4-3} = 6a^{-7}$

N/B: $\frac{1}{a^2} = 1 \times a^{-2} = a^{-2}$

2. $\frac{1}{a^{-2}} = 1 \times a^2 = a^2$

3. $\frac{1}{3^2} = 1 \times 3^{-2} = 3^{-2}$

4. $\frac{1}{3^{-2}} = 1 \times 3^2 = 3^2$

5. $\frac{9}{3a^{-2}} = \frac{9}{3} \times a^2 = 3 \times a^2 = 3a^2$

6. $\frac{9}{3a^2} = \frac{9}{3} \times a^{-2} = 3a^{-2}$

7. $\frac{4}{2a^2} = \frac{4}{2} \times a^{-2} = 2 \times a^{-2} = 2a^{-2}$

8. $\frac{4}{2a^{-2}} = \frac{4}{2} \times a^2 = 2 \times a^2 = 2a^2$

9. $\frac{4a^{-6} \times 2a^2}{2a^{-3}} = \frac{4 \times 2 \times a^{-6} \times a^2}{2 \times a^{-3}}$
$= \frac{8 \times a^{-4}}{2 \times a^{-3}} = 4 \times a^{-4} \times a^3$
$= 4 \times a^{-4+3} = 4a^{-1}$

10. $\frac{2a^{-2} \times 6a^4}{3a} = \frac{2 \times 6 \times a^{-2} \times a^4}{3 \times a}$
$= \frac{12a^{-2+4}}{3 \times a^1} = \frac{4a^2}{a^1} = 4a^2 \times a^{-1}$
$= 4a^{2-1}$
$= 4a^1 = 4a$

11. $3a^2 b \times 4a^3 b^4 = 3 \times 4 \times a^2 \times a^3 \times b^1 \times b^4$
$= 12a^5 b^5$

12. $3a^{-3} b^2 \times 5a^{-2} b^{-4} = 3 \times 5 \times a^{-3} \times a^{-2} \times b^2 \times b^{-4}$
$= 15a^{-5} b^{-2}$

13. $\underline{3a^2 b \times 6a^3 b^4} = \underline{3 \times 6 \times a^2 \times a^3 \times b \times b^4}$

$$= \frac{18a^5b^5}{2 \times a \times b^2} = \frac{9 \times a^5 \times a^{-1} \times b^5 \times b^{-2}}{2ab^2 \quad 2 \times a \times b^2}$$

$= 9a^4 b^3$

Exponential Equations:

N/B: 1. $4 = 2^2$

3. $8 = 2 \times 2 \times 2 = 2^3$
4. $32 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$
5. $64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$
6. $25 = 5 \times 5 = 5^2$
7. $125 = 5 \times 5 \times 5 = 5^3$
8. $16 = 4 \times 4 = 4^2$
9. $64 = 4 \times 4 \times 4 = 4^3$
10. $9 = 3 \times 3 = 3^2$
11. $27 = 3 \times 3 \times 3 = 3^3$
12. $81 = 9 \times 9 = 9^2$
13. $81 = 3 \times 3 \times 3 \times 3 = 3^4$

Q1. If $2^x = 8$, find x.

Soln.

Since $8 = 2 \times 2 \times 2 = 2^3$, then $2^x = 8 \Rightarrow 2^x = 2^3 \Rightarrow x = 3$.

Q2. Given that $2^{x+1} = 16$, find x.

Soln.

$16 = 2 \times 2 \times 2 \times 2 = 2^4$

$\therefore 2^{x+1} = 16 \Rightarrow 2^{x+1} = 2^4$

$\Rightarrow x + 1 = 4, \Rightarrow x = 4 - 1 = 3.$

Q3. If $4^{2x-1} = 64$, find x.

Soln.

$64 = 4 \times 4 \times 4 = 4^3$

$\therefore 4^{2x-1} = 64$

$\Rightarrow 4^{2x-1} = 4^3$

$\Rightarrow 2x - 1 = 3,$

$\Rightarrow \; 2x = 3 + 1 = 4.$

$\therefore x = {}^{4}/_{2} = 2.$

Q4. Given that $5^{2(n-1)} = 125$, find n.

Soln.

$125 = 5 \times 5 \times 5 = 5^3$

$\therefore 5^{2(n-1)} = 125$

$\Rightarrow 5^{2(n-1)} = 5^3$

$\Rightarrow 2(n - 1) = 3, \Rightarrow 2n - 2 = 3,$

$\therefore 2n = 3 + 2 \Rightarrow 2n = 5,$

$\therefore n = {}^{5}/_{2} = 2.5.$

Q5. Given that $2^{x+1} = 8^x$, find x.

Soln.

$2^{x+1} = 8^x$, but $8 = 2^3$

$\Rightarrow 2^{x+1} = 2^{3x}$

$\Rightarrow x + 1 = 3x, \Rightarrow 1 = 3x - x$

$\Rightarrow 1 = 2x,$

$\Rightarrow x = {}^{1}/_{2}$ or $x = 0.5$

N/B: If there is a plus or a minus sign between a number and a letter, they must be placed within a bracket before a number can be used to multiply them.

Q6. If $3^n = 81^{n-1}$, find the value of n.

Soln.

$81 = 3^4$

$\therefore 3^n = 81^{n-1} \Rightarrow 3^n = 3^{4(n-1)}$

$\Rightarrow n = 4(n - 1),$

$\Rightarrow n = 4n - 4$

$\Rightarrow n + 4 = 4n,$

$\Rightarrow 4 = 4n - n \Rightarrow 4 = 3n,$

$\Rightarrow n = {}^{4}/_{3} = 1.3.$

Q7. If $3^{2(x-1)} = 27^{x+2}$, find the value of x.

Soln.

$27 = 3^3$

$\therefore 3^{2(x-1)} = 27^{x+2}$

$\Rightarrow 3^{2(x-1)} = 3^{3(x+2)}$

=> 2(x – 1) = 3(x + 2),

=> 2x – 2 = 3x + 6,

=> -2 = 3x + 6 – 2x,

=> -2 – 6 = 3x – 2x

=> - 8 = x,

∴ x = - 8.

Q8. Given that $(^1/_2)^{n+1} = {}^1/_4$, determine the value of n.

Soln.

$^1/_4 = (^1/_2)^2$

$\therefore (^1/_2)^{n+1} = ¼$

$\Rightarrow (^1/_2)^{n+1} = (^1/_2)^2$

=> n + 1 = 2, => n = 2 – 1

=> n = 1.

Q9. If $(^1/_3)^n = {}^1/_9$, find n.

Soln.

Since $^1/_9 = (^1/_3)^2$

$\Rightarrow (^1/_3)^n = {}^1/_9 =$

$(^1/_3)^n = (^1/_3)^2 \Rightarrow n = 2.$

Q10. Given that $(^1/_3)^{n-1} = {}^1/_{27}$, evaluate n.

Soln.

$^1/_{27} = (^1/_3)^3$

$(^1/_3)^{n-1} = {}^1/_{27}$

$\Rightarrow (^1/_3)^{n-1} = (^1/_3)^3$

=> n – 1 = 3 => n = 3 + 1 = 4

∴ n = 4.